Feature Lesson: Disc/Washer Method

Class: BC and AB Calculus Grade level 12
Subject: Application of the Definite Integral
Lesson Title: Disc/Washer Method
Lesson Length: 2.5 days on block schedule

The purpose of this lesson is to help students learn the Disc or Washer Method to calculate the volume of a solid by rotating a region about an axis of rotation using “slices” perpendicular to the axis of rotation.

Content Objectives: All students will learn how and when to apply the Disc/Washer Method to find volumes of solids.

1. ELL students become familiar with sketching graphs and shade the appropriate regions with the aid of a graphing calculator.
2. ELL students will demonstrate labeling the outer radius and inner radius dependent on graphs.
3. ELL students will “set-up” an integral to find the requested volume.

Language Objectives: All students will be able to verbally explain the process of calculating volumes by the Disc or Washer method with appropriate terminology.

1. ELL students will read math examples to determine the axis of rotation.
2. ELL students will discuss how to determine and label the outer radius and inner radius.
3. ELL students will write the main points/summarize steps to find the requested volume.

Culture Objectives:
1. ELL students will find a real-life object created in their country that could be created by the Disc/Washer method – example: a bowl. They will share and discuss this with the class.

North Carolina Standard Course of Study High School Math Goals:
Advanced Placement Calculus competency goal 3.04: Define and use appropriate integrals in a variety of applications...Find the volume of a solid.
Objective is also listed in the College Board’s curriculum guide for AB Calculus, part III – applications of integrals.
Teacher Materials:  
textbook: *Single Variable Calculus 5e* by James Stewart  
prepared Notebook lesson for *Smart* board  
computer  
projector  
Internet access  
software package: *Calculus in Motion™*  
paper/pencils/colored pencils  
graphing calculator  
wood manipulatives  
tape  
headphones  
*Smart* board airliner

Activity 1: Visualization of Disc/Washer method

Visual Aids will be used in the introduction to this lesson as well as continually through the unit. These aids will assist students by allowing students to “see” what is happening three-dimensionally. Students will participate with their usage in whole group lessons as well as in cooperative groups. I will encourage ELL students to use the visual aids as much as possible during this lesson. I will constantly refer to these visual cues as I teach/explain methods.

1) Provide a slip of paper (1 in by 8 in) and piece of tape to each student. Students will tape the 1 inch side perpendicular to the pencil. Students will spin the pencil to see the paper revolving around the “axis of symmetry” and producing a 3-D disc.

2) Websistes will be visited to demonstrate objective.

3) The interactive software package will be used with specific examples to demonstrate objectives as students copy pre-selected examples in their notes. All students are encouraged to use this software on my computer. I will specifically encourage the ELL students to use this technology.

4) I will use a yardstick (“slice”) and my body (“axis of rotation”) to demonstrate that the slices must be perpendicular to the axis of rotation to produce discs.

5) Students may use the graphing calculator to assist sketching graphs.

6) Wood manipulatives will be available to students where they can hold and study the models.

Activity 2: Notes on Disc/Washer method

I will provide a list of vocabulary words/phrases that is used in this lesson. I will encourage ELL students to write this list in their notes. Students will copy pre-selected examples in their notebooks. I expect my students to sketch the graph as well as the “set-ups” in their notebook. I will employ redundancies and verbal repetitions during the whole-group lesson while students copy the mathematics in their notebook. As I discuss the examples, I will encourage students to
Activity 2 continued:
use multiple colors to label the outer and inner radii. I will use the Smart board airliner as I teach so that I can walk around the classroom. I will observe the ELL notes to be sure they have not missed anything. When students go home with practice problems, I will encourage students to visit the website http://www.hippocampus.org. This site provides explanations in English as well as Spanish to provide additional home support. Students are also allowed to use this during class time if desired. I have a set of head phones that can be used so as to not disrupt other students while listening. ELL students will be encouraged to use this website.

Activity 3: Collaborative Grouping/Conclusion

With teacher led verbal directions, groups of three will be assigned so that there are varying levels of abilities that will foster collaborative communication among students. Many times, our ELL students are Hispanic. If so, I will place ELL students in a group with another student who is learning the Spanish language. This encourages learning and mathematical discourse in both languages which benefits all students in the group. First, I will ask students to organize the steps to solve problems in small sequential steps and define desired lengths. Students can do this in English as well as their native language. I will assign alternating roles: “sketcher”, “labeler”, and “setter upper” for each group to use while practicing problems. As problems are being practiced, students take on one of these roles. For the next practice problem, they alternate roles. They continue this alternating pattern for the remainder of the practice problems. All students, especially ELL students, will benefit from these roles because it allows multiple opportunities to verbalize the methods, listen to other students explain procedures, produce visuals and write correct mathematical notations that will assist in their learning of the objectives. All groups are allowed to use the interactive software on my computer to gain a better understanding of the visualization of the problems. I will continually monitor groups for understanding, correct mathematical notations and explanations of the objectives. I will use praise and encouragement and constructive feedback as I monitor the groups.

Activity 4: Assessment/Quiz time

First, their assigned groups will take an alternative assessment on small whiteboards that I can easily check for accuracy as I walk around the room. This also encourages students to verbally discuss methods, visually see the objects being generated and to use proper mathematical written language to write their results. This allows ELL students to again practice what they have learned in a stress-free activity. After each group completes this activity, students leave the group and independently take the quiz with paper/pencil so that I can learn which students understand and not understand yet the objectives. Directions will be read verbally before the quiz starts and extra time will be available if needed for ELL students to complete the quiz. The quiz and grading rubric is attached. If needed, a modified grading rubric will be used for ELL students. At this level of mathematics, this particular modification is usually not needed. If other modifications are in place during the learning of the objectives, students generally understand the material quite well.
Instructional Technology Used for Teaching and Learning:

By allowing students access to the above mentioned visuals: Interactive Internet websites, interactive software, Smart board and graphing calculators, great learning can take place. I use the Internet, interactive software and Smart board in my lesson for visualizations (level 3). I allow my students to use the computer to visit websites and especially interact with the software package (level 6). Students are able to see the mathematics take place in 3-D. They see the object as it is being created. They are allowed to create their own problems and answer the “what ifs” that students so often have (level 5). The http://www.hippocampus.org website (level 7) is great for all students to use, especially ELL Hispanic students. It explains how to do problems and shows how to write correct mathematical notations. Each student has a graphing calculator so they can continue the use of this technology at home (level 6). This is especially good if they do not have Internet access at home. These methods improve students’ self-confidence in their ability to succeed in this unit. Students can take ownership of their learning. The instructional level of technology would range from level 3 to level 7 as noted above depending on the activity.

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Permission to Publish: Yes
Find the volume of the solid generated by revolving about the given axis bounded by the following curves using the DISK/WASHER method.

1. (SOLVE COMPLETELY) about x-axis: \( y = x^2, \ x = 2, \ y = 0 \)

   Just set up the rest

2. about x-axis: \( y = x^2, \ y = 2x \)

3. about y-axis: \( x = 1 + y^2, \ y = x - 3 \)

4. about y-axis: \( y = x^2, \ y\)-axis, \( y = 4 \)

5. about \( y = -1 \): \( y = x^2 + 1, \ y = 9 - x^2 \)

6. about \( x = 5 \): \( y = x^4, \ y = 16, \ x = 0 \)
Find the volume of solid generated by revolving about the given axis bounded by the following curves using the DISK/WASHER method.

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\[
\pi \int_{0}^{\frac{3\sqrt{5}}{5}} (x^2)^2 \, dx = \pi \left[ \frac{5}{5} \right] = 5 \text{ pts additional}
\]

2. about x-axis: \( y = x^3, \ y = 2x \)

3. about y-axis: \( x = 1 + y^2, \ y = x - 3 \)

4. about y-axis: \( y = x^2, \ y = 4 \)

5. about \( y = -1 \): \( y = x^2 + 1, \ y = 9 - x^2 \)

6. about \( x = 5 \): \( y = x^4, \ y = 16, \ x = 0 \)